HALE SCHOOL

Semester Two Examination, 2019

Question/Answer booklet

Year 11

MATHEMATICS METHODS UNITS 1 AND 2

Section Two: Calculator-assumed

Circle your teacher's initials

IFB MS JIB STL SAV

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	90	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed **65% (90 Marks)**

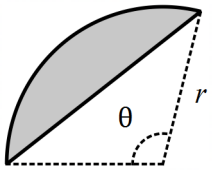
This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

(a) Convert $96°$ to an exact radian measure. (1 mark)

(b) A segment of a circle of radius 33 cm is shown below, where $\theta = 96°$.



(i) Determine the area of the segment. (2 marks)

(ii) Determine the perimeter of the segment. (3 marks)

Question 10 **(3 marks)**

Find the size of angle Q in triangle PQR given that angle P measures 56°, PR = 13.5 metres and QR = 16.8 metres.

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Question 11 **(7 marks)**

From a random survey of telephone usage in 261 households it was found that 155 households had access to both mobiles and landlines, 54 households had no access to a mobile and 145 more households had landlines than did not.

(a) Complete the missing entries in the table below. (3 marks)

	Mobile	No mobile	Total
Landline	155		
No landline			
Total			261

(b) If one household is randomly selected from those surveyed, determine the probability that

(i) it had access to a mobile phone. (1 mark)

(iii) it had access to a mobile given that it no access to a landline. (1 mark)

(c) Use your answers from part (b) to comment on the possible independence of households having access to a landline and households having access to a mobile phone. (2 marks)

Question 12 **(10 marks)**

When a manufacturer makes x litres of a chemical using process B, the cost in dollars per litre $C(x)$ varies according to the rule

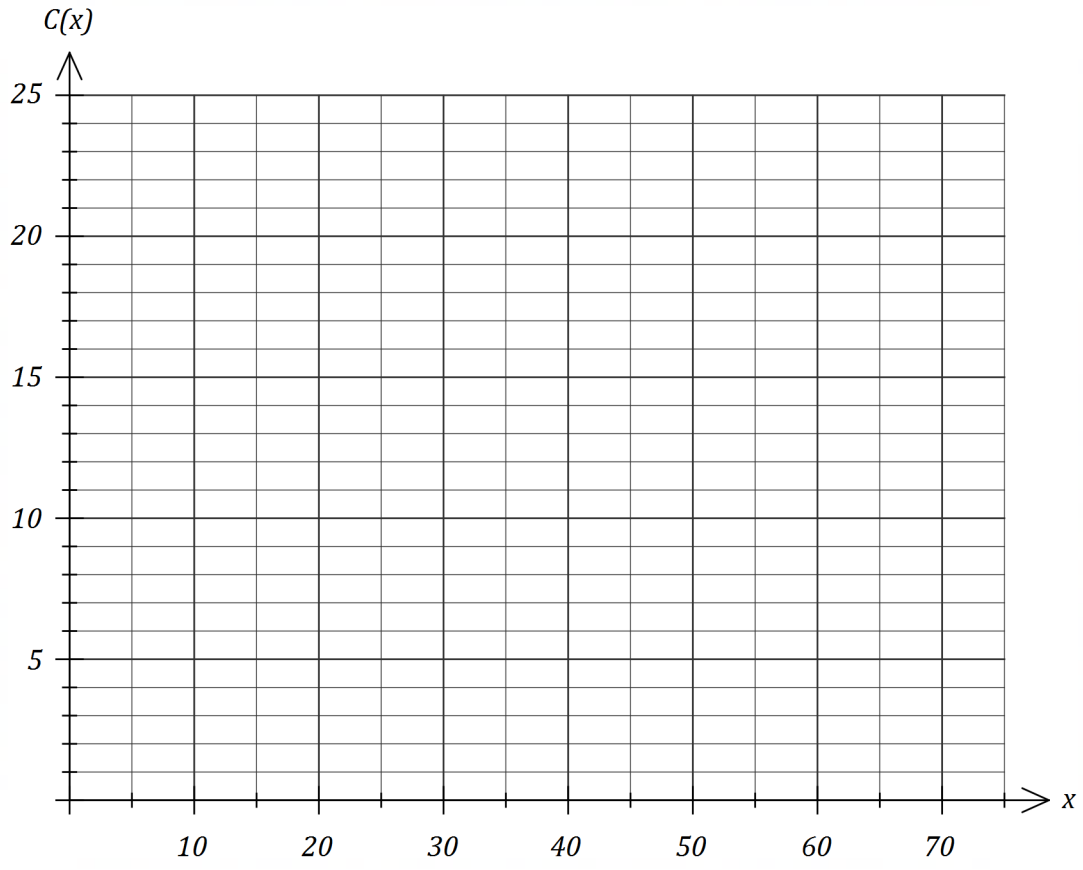
$$C(x) = \frac{300}{x + 10}, \qquad 5 \leq x \leq 65.$$

(a) Determine

(i) the cost per litre when 38 L is made. (1 mark)

(ii) the total cost of making 14 L of the chemical. (2 marks)

(b) Graph the cost per litre over the given domain on the axes below. (3 marks)



(c) State the range of $C(x)$. (1 mark)

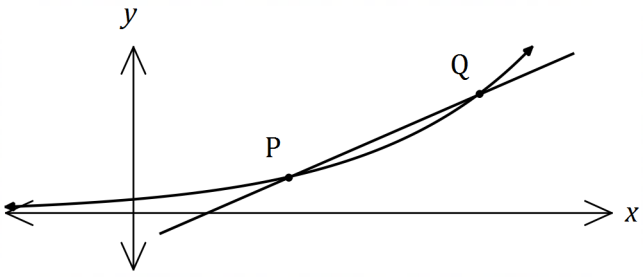
When the manufacturer uses process D, the cost in dollars per litre $K(x)$ is modelled by

$$K(x) = \frac{35}{2} - \frac{x}{4}, \qquad 5 \leq x \leq 65.$$

(d) Add this function to the graph and hence determine the production quantities for which process B is cheaper than process D. (3 marks)

Question 13 **(5 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = 4^x$, together with the secant to the curve through the points P and Q.



P has coordinates $(1, 4)$ and Q has coordinates $\big(1 + h, f(1 + h)\big)$ where $0 < h \leq 1$.

(a) Show that the gradient of $PQ = 12$ when $h = 1$. (1 mark)

(b) Complete the second column in the table below, rounding values to 4 decimal places. (2 marks)

h	$\frac{f(1+h) - f(1)}{h}$
1	**12**
0.1	
0.01	
0.001	

(c) Determine an estimate, correct to 3 decimal places, for the value that $\frac{f(1+h) - f(1)}{h}$ approaches as h becomes closer and closer to 0 and state what this value represents. (2 marks)

Question 14 **(5 marks)**

A geometric sequence has a second term of -8.4 and a sum to infinity of 15.

Determine the sum of the first 4 terms of the sequence.

See next page

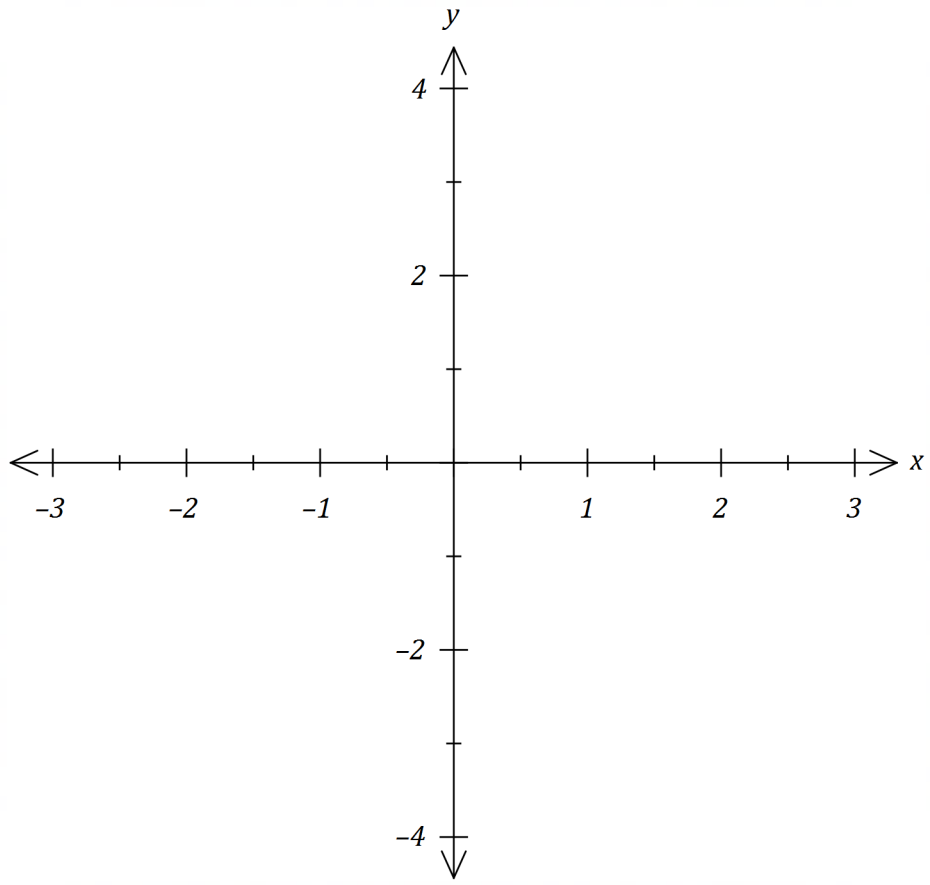
Question 15 **(12 marks)**

The function f is defined by $f(x) = -3x^3 + px^2 + qx + r$, where p, q and r are constants.

The graph of $y = f(x)$ has the following features:

- passes through $(0, -3)$ and $(-1, 0)$
- has a local maximum at $(1, 0)$

(a) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)



(b) Determine the value of p, the value of q and the value of r. (3 marks)

(c) Use a calculus method to determine the exact coordinates of the local minimum of the graph of $y = f(x)$. Justifying the nature of turning point is not necessary. (3 marks)

(d) Determine the coordinates of the point where the tangent to $y = f(x)$ at $(0, -3)$ intersects the curve $y = f(x)$, other than at the point of tangency. (3 marks)

See next page

Question 16 **(7 marks)**

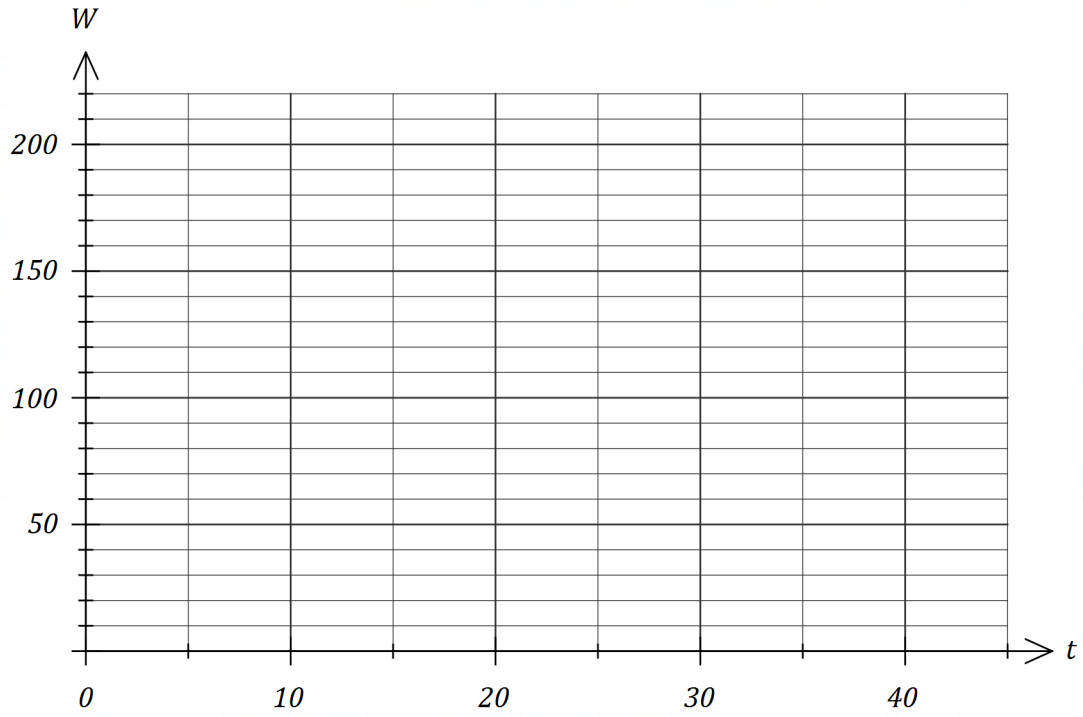
The amount of water in a tank, W litres, varies with time t, in minutes, and can be modelled by the equation $W = 200 - 185(1.2)^{-t}$, $t \geq 0$.

(a) Determine amount of water in the tank

(i) initially. (1 mark)

(ii) after 15 minutes. (1 mark)

(b) Graph W against t for $0 \leq t \leq 45$ on the axes below. (3 marks)



(c) Over time, the amount of water in the tank approaches v litres. State the value of v and the value of t at which the amount of water in the tank reaches 99% of v.

(2 marks)

Question 17 **(7 marks)**

When a patient takes a painkilling drug D, the probability that they experience some side effects is known to be 0.2.

(a) A doctor prescribes drug D to two unrelated patients. Determine the probability that

(i) neither patient experiences some side effects. (1 mark)

(ii) one patient experiences some side effects and the other does not. (2 marks)

Other painkilling drugs are available. Of those who take drug D, 75% of patients who suffer some side effects will switch to another drug whereas no patient who has no side effects will switch.

(b) The doctor prescribes drug D to a patient. Determine the probability that the patient does not switch to another drug. (2 marks)

(c) The doctor prescribes drug D to three unrelated patients. Determine the probability that at least one of these patients switch to another drug. (2 marks)

See next page

Question 18 **(8 marks)**

Two events A and B are such that $P(A \cap \bar{B}) = x$, $P(A) = 0.2$ and $P(\bar{A} \cap B) = 0.6$.

(a) Determine $P(A \cap B)$ when $x = 0.12$. (2 marks)

(b) Determine an expression for $P(A \cap B)$ in terms of x. (1 mark)

(c) Determine the value of x when

(i) A and B are mutually exclusive. (1 mark)

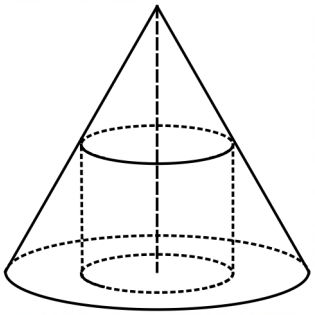
(ii) A and B are independent. (2 marks)

(iii) $P(A|B) = 0.04$. (2 marks)

See next page

Question 19 **(7 marks)**

A right circular cone of base radius 8 cm and height 24 cm stands on a horizontal surface. A cylinder of radius x cm and volume V cm^3 stands inside the cone with its axis coincident with that of the cone and such that the cylinder touches the curved surface of the cone as shown.



(a) Show that $V = 24\pi x^2 - 3\pi x^3$. (3 marks)

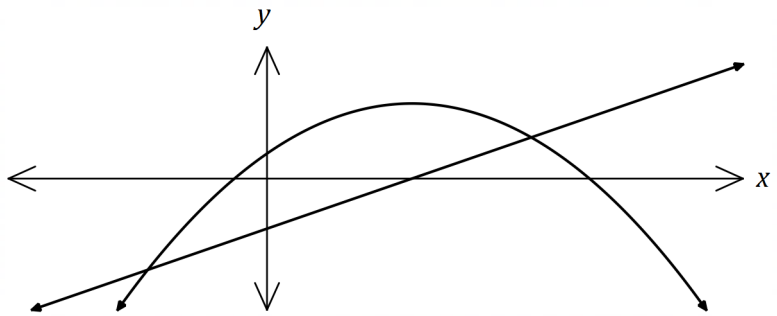
(b) Given that x can vary, use a calculus method to determine the value of x for which V is a maximum.

(4 marks)

See next page

Question 20 **(6 marks)**

The graphs of $y = f(x)$ and $y = g(x)$ are shown below where $f(x) = 1 + 4x - 2x^2$ and $g(x) = 2x + k$.



Determine the value(s) of the constant k so that the equation $f(x) = g(x)$ has

(a) one solution. (5 marks)

(b) no solutions. (1 mark)

Question 21 **(7 marks)**

A fair four-sided dice numbered $1, 2, 3$ and 4 is thrown n times until it lands on a 4.

(a) Show that the probability that $n = 2$ is $\frac{3}{16}$. (1 mark)

(b) Determine the probability that $n = 5$. (1 mark)

(c) The probabilities form a geometric sequence. Write an expression in terms of n for the probability that the first 4 is thrown on the n^{th} throw. (1 mark)

(d) Determine the probability that the first 4 is thrown in 7 or less attempts. (2 marks)

(e) The probability that the first 4 is thrown in k or less attempts must be at least 99.5%. Determine the least value of integer k. (2 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____